17th International Colloquium
SUSTAINABLE DECISIONS IN BUILT ENVIRONMENT
15 May 2019, Vilnius, Lithuania
Vilnius Gediminas Technical University
e-ISBN 978-609-476-195-9
Article ID: colloquium.2019.002
https://doi.org/10.3846/colloquium.2019.002

A statistical approach for improvement of Best Worst Method (BWM)

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Abstract. This paper endeavors to measure the consistency of a decision-making tool, popularly known as Best Worst Method (BWM), which is one of the latest developments in multiple-criteria decision analysis (MCDA). BWM is finding a vast arra of applications in the literature. Several investigators have extended this tool. BWM measures the weight of decision-making criterion and is recognized as a subjective decision tool. The first step in this method is to find the best and worst criterion, while we suppose several experts are asked to present their evaluation over set of criteria. The aim is to measure how these judgments are consistent and reliable. So, we statistically (using $\chi 2$ distribution) add a pre-evaluation to experts' opinion and analyze whether the agreement of experts' opinions is satisfactory and group opinion is established. This action improves the quality of the decision-making process by incorporating the reliability evaluation of experts' idea. This extension for BWM helps decision makers in facilitating and getting results that are more consistent for criteria evaluation. We present examples in sustainable construction and architecture project.

Keywords: Best Worst Method, group decision, consistecny, multi criteria decision analysis.

Introduction

Improving and extending decision-making tools is very popular among the academic community and researchers. Range of studies have been conducted to show the quality of decision-making increases when the consistency improves. This issue in multi attribute decision making is highlighted since decades. The fundamental complexity in many decision making condition is the theme of consistency and reliability (Leung & Cao, 2000). One method that was under investigation many times is analytical hierarchy procees (AHP) (Alonso & Lamata, 2006; Xu, 2000; Aguarón, Escobar, & Moreno-Jiménez, 2016). Like AHP which is a subjective decision making tool, (Rezaei, 2015) invented a new algorithm that operates based on a linear programing model. The method is named Best Worst Method (BWM) and practically due to its effective nature ereceived too much attentions in many fields and disciplines (Gupta & Barua, 2016; Rezaei, Nispeling, Sarkis, & Tavasszy, 2016; Rezaei, Wang, & Tavasszy, 2015; Ahmadi, Kusi-Sarpong, & Rezaei, 2017).

Recently Zolfani, Yazdani, and Zavadskas (2018) worked on an extended version of stepwise weight assessment ratio analysis method. The authors used a statistical technique in order to test whether the experts agree on their judgments. The approach confirms the consistency of the method and allows decision makers go forward and rely on the results. In this paper we are going to apply the recent approach for BWM in order to check its usability and performance. Second section presents the algorithm, then an example about evaluating sustainable building factors are provided and a conclusion ends the debate.

1. Improved BWM method

Best Worst Method (BWM) is one of the efficient and recent born MCDA tool. Decision makers usually utilize it for determining the subjective weight of criterion. The method has captured many attentions in various applications as

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supplier selection, energy resources and air transportation quality. BWM is used to produce the importance of decision criteria based on expert (decision makers, respondents) attitude and in the classical version of BWM method there is no mechanism to test expert's attitude consistency. We suppose several experts are willing to present their judgment on some factors. How then all these judgments can be trustable and reliable is the question of this paper. In other words, we try to figure out how the results of aggregated opinions are consistent. The experts) are asked to evaluate each criterion based on their cognition, experience and knowledge and to rank them in order of their preferences (from best to worst). Then, we follow the following steps:

Step 1: *Calculation of the average criterion values* $\overline{t_j}$:

$$\overline{t_j} = \frac{\sum_{k=1}^{t} t_{jk}}{r}, \tag{1}$$

where t_{jk} indicates the ranking of the j^{th} criteria by the k^{th} respondent and r is the number of respondents.

Step 2: *Calculation of criteria weights* (q_j).

The criteria weights are calculated by dividing the average value of each criterion by the sum of the criteria priority (rank) values (t_j):

$$q_j = \frac{\overline{t_j}}{\sum_{j=1}^{n} t_j}. \tag{2}$$

The total criteria weight must be equal to one, signifying $\sum_{j=1}^{n} q_j = 1$ and t_j is the criteria priority values.

Step 3: *Verifying the reliability of the expert opinions*.

Step **3-1:** Now, for the purpose of verifying the reliability of the expert opinions, dispersion in criteria ranking as given by the experts is first computed using Eq. (3), followed by the calculation of the variation of the obtained values using Eq. (4). **Dispersion** basically indicates the measurement of the variation between the multiple expert opinions.

$$\sigma^2 = \frac{1}{r-1} \sum_{k=1}^{n} \left(t_{jk} - \overline{t_j} \right)^2 \tag{3}$$

$$\beta_j = \frac{\sigma}{\overline{t_j}} \tag{4}$$

Step **3-2:** Determination of coefficient of concordance (agreement) for the experts' opinions:
Determine the coefficient of concordance (W) of the experts or the respondents (eleven for the illustrative case study) opinions to express the reliability of individual expert opinion using Eq. (5).

$$W = \frac{12S}{r^2\left(n^3 - n\right) - r\sum_{k=1}^{r} T_k}. \tag{5}$$

$$W \in [0,1],$$

where S is the total square deviation of the rankings of each criterion, expressed by Eq. (6).

$$S = \sum_{j=1}^{n} \left[\sum_{k=1}^{r} t_{jk} - \frac{1}{n} \sum_{j=1}^{n} \sum_{k=1}^{r} t_{jk} \right]^2. \tag{6}$$

In this equation, T_k the index of reiterated ranks in the r rank, r is the number of respondents and n is the number of criteria. However, as the calculated W value is stochastic; thus, significance of the concordance coefficient should be estimated. Kendall (1970) indicated that when n is greater than 7, a distribution with the degrees of freedom $v = n - 1$ should be considered by the experts or the respondents (eleven for the illustrative case study).

Step **3-3:** Calculation of χ^2:

$$\chi^2_{\alpha,v} = W.r.(n-1). \tag{7}$$

Step **3-4:** Testing the $\chi^2 > \chi^2_{table}$. It has been proved that if the calculated χ^2 value is greater than the critical tabular value χ^2_{table} for the pre-selected level of significance, then the hypothesis about the agreement of independent experts 'judgments' is not rejected. If $\chi^2 > \chi^2_{table}$ the significance of concordance coefficient exists on α level, then the agreement of experts 'opinions is satisfactory and group opinion is established. In this case, the tabular value (χ^2_{table}) was taken from Fisher and Yates statistical tables (Fisher & Yates, 1963). Once the agreement among expert opinion is achieved, then the mains steps for BWM is denoted:

Step 4: The decision maker (DM) determines a Set of decision criteria: $\{c_1, c_2, \ldots, c_n\}$.

Step 5: The DM chooses the best and the worst criteria. In this step, the DM chooses the best and the worst criteria among the set of identified criteria in last step. The best criterion represents the most desirable or the most significant one while the worst criterion is the least important one among others.

Step 6: The DM conducts pairwise comparisons between the best criterion and the other criteria. In this step, the goal is to identify the preference of the most important criterion to the other criteria. DM uses a scale from 1 to 9 (1: equally important, and 9: extremely more important). The comparison outcome is described as Best-to other vector: $A_B = (a_{B1}, a_{B2}, \ldots, a_{Bn})$. Where a_{Bj} represents the preference of the best criterion B over the criterion j and $a_{BB} = 1$

Step 7: The DM conducts pairwise comparison between the other criteria and the worst criterion. Same as last step, the comparison results are expressed by Other-to-worst vector: $A_W = (a_{1W}, a_{2W}, \ldots, a_{nB})^T$. Where a_{jw} represents the preference of the best criterion j over the criterion W and $a_{WW} = 1$

Step 8: Calculating the optimal weights: $\left(W_1^*, W_2^*, \ldots, W_n^*\right)$, For more information of this method, refer to (Rezaei, 2015).

2. An example of the proposed technique

In this section, an empirical example of evaluating factors in a sustainable building has been presented in order to establish the appropriateness of the adopted statistical approach for measuring the consistency of BWM. As the table shows, Eleven experts participated in this research considering these elements: Technical factors (C_1), waste disposal system (C_2), Environmental factors (C_3), Total costs of project (C_4), Safety and security factors (C_5), green or sustainable materials (C_6) and energy consumption control (C_7). The experts were demanded to rate each factor from 1 to 7. In case of BWM method, suppose each expert provides worst and best criterion. For example, Expert 1 considers C_3 as best option, and then he/she compares other criteria to that (for example C_1 6 times greater than C_3). We call this table (Table 1) efficiency rank of attributes. The next step is to find the attribute weights using Eq. (2). As seen in Table 2, all the computation are observed.

Table 1. Expert primary judgment over seven factors (Attribute efficiency)

Experts	C_1	C_2	C_3	C_4	C_5	C_6	C_7	
1	6	2	1	3	4	5	7	
2	6	1	3	2	5	4	7	
3	5	1	2	4	3	7	6	
4	7	2	1	4	3	6	5	
5	6	1	3	4	2	7	5	
6	4	2	3	5	1	7	6	
7	5	1	2	3	4	6	7	
8	5	1	3	4	2	7	6	
9	5	1	2	4	3	7	6	
10	4	3	1	5	2	7	6	
11	5	1	2	4	3	7	6	
Sum	**58**	**16**	**23**	**42**	**32**	**70**	**67**	Total = 308, Av= **44**
Average (Equation 1)	5.27	1.45	2.09	3.82	2.91	6.36	6.09	
Attribute Priorities	5	1	2	4	3	7	6	
Attribute weights (Equation 2)	0.188	0.052	0.075	0.136	0.104	0.227	0.218	

While the value of χ^2_{table} (Table in appendix), for (v = 6) and importance level of 1% is equal to 16.81. Then, because 54 is bigger than 16.81, the results are consistent and acceptable. Therefore, the experts can continue to compute the rest of the process.

The weights are appeared in Table 3 and consistency 0.19 shows to what extent the results are reliable.

Table 2. Expert primary judgment over seven factors

Process of computation	Efficiency attributes xi; j = 1, 2, …						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
Sum of ranks	58	16	23	42	32	70	67
The average attribute **rank value**	5.273	1.455	2.091	3.818	2.909	6.364	6.091
Attribute rank	5	1	2	4	3	7	6
Attribute weight	0.188	0.052	0.075	0.136	0.104	0.227	0.218
Sum $\sum_{k=1}^{n}\left(t_{jk}-\bar{t}_j\right)^2$	8.18	4.73	6.91	7.64	12.91	10.55	4.91
Dispersion of expert σ^2	0.82	0.47	0.69	0.76	1.29	1.05	0.49
Variation $\beta_j = \frac{\sigma}{\bar{t}_j}$	0.172	0.473	0.398	0.229	0.391	0.161	0.115
Ranking sum average	**44**						
The total square ranking deviation	S = 2774						
The coefficient of concordance (**W**)	**W = 0.82**						
The significance of the concordance coefficient (no related ranks) Where $\frac{1}{n-1}\sum_{k=1}^{r}T_k = 0$	$\chi^2_{\alpha,v} = \frac{12*2774}{11*7(8)} = 54$						
Rank of table concordance (χ^2_{table}) when the importance equal to 1%.	The freedom degrees value of a solved problem, v = n–1 = 6						
Compatibility of expert judgement $\chi^2_{\alpha,v} = 54 > \chi^2_{table}$	– The hypothesis about the consent of experts in rankings is accepted						

Table 3. The weights of BWM

Factors	technical factors	waste disposal	Env. Factors	total cost	safety & security	green materials	energy control
Weights	0.0859	**0.3239**	0.1718	0.1718	0.1288	0.0442	0.0736

Conclusions

In this paper, an improved version of the original BWM method has been proposed. It has been shown that adding an extra statistical algorithm at the beginning of the calculation of BWM is essential in order to check the accuracy of group decision makers. This strategy might be useful in other subjective criteria estimation tools like Decision making trial and evaluation laboratory (DEMATEL), Entropy, criteria importance through inter criteria correlation (CRITIC) methods etc.

Acknowledgements

The authors are thankful of being invited for this communication and another thanks to reviewers.

References

Aguarón, J., Escobar, M. T., & Moreno-Jiménez, J. M. (2016). The precise consistency consensus matrix in a local AHP-group decision making context. *Annals of Operations Research*, *245*(1-2), 245-259. https://doi.org/10.1007/s10479-014-1576-8

Ahmadi, H. B., Kusi-Sarpong, S., & Rezaei, J. (2017). Assessing the social sustainability of supply chains using Best Worst Method. *Resources, Conservation and Recycling*, *126*, 99-106. https://doi.org/10.1016/j.resconrec.2017.07.020

Alonso, J. A., & Lamata, M. T. (2006). Consistency in the analytic hierarchy process: a new approach. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, *14*, 445-459. https://doi.org/10.1142/S0218488506004114

Fisher, R. A., & Yates, F. (1963). *Statistical tables for biological, agricultural and medical research* (6^{th} ed.). London: Oliver and Boyd.

Gupta, H., & Barua, M. K. (2016). Identifying enablers of technological innovation for Indian MSMEs using best–worst multi criteria decision making method. *Technological Forecasting and Social Change*, *107*, 69-79. https://doi.org/10.1016/j.techfore.2016.03.028

Kendall, M. G. (1970). *Rank correlation methods* (4^{th} ed.). Griffin, London.

Leung, L. C., & Cao, D. (2000). On consistency and ranking of alternatives in fuzzy AHP. *European Journal of Operational Research*, *124*, 102-113. https://doi.org/10.1016/S0377-2217(99)00118-6

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega*, *53*, 49-57. https://doi.org/10.1016/j.omega.2014.11.009

Rezaei, J., Nispeling, T., Sarkis, J., & Tavasszy, L. (2016). A supplier selection life cycle approach integrating traditional and environmental criteria using the best worst method. *Journal of Cleaner Production*, *135*, 577-588. https://doi.org/10.1016/j.jclepro.2016.06.125

Rezaei, J., Wang, J., & Tavasszy, L. (2015). Linking supplier development to supplier segmentation using *Best Worst Method. Expert Systems with Applications*, *42*(23), 9152-9164. https://doi.org/10.1016/j.eswa.2015.07.073

Xu, Z. (2000). On consistency of the weighted geometric mean complex judgement matrix in AHP. *European Journal of Operational Research*, *126*(3), 683-687. https://doi.org/10.1016/S0377-2217(99)00082-X

Zolfani, S. H., Yazdani, M., & Zavadskas, E. K. (2018). An extended stepwise weight assessment ratio analysis (SWARA) method for improving criteria prioritization process. *Soft Computing*, *22*(22), 7399-7405. https://doi.org/10.1007/s00500-018-3092-2

Appendix

	α					
v	0.100	0.050	0.025	0.010	0.005	0.001
1	2.7055	3.8415	5.0239	6.6349	7.8794	10.8276
2	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155
3	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662
4	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668
5	9.2364	11.0705	12.8325	15.0863	16.7496	20.5150
6	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577
7	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219
8	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245
9	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772
10	15.9872	18.3070	20.4832	23.2093	25.1882	29.5883
11	17.2750	19.6751	21.9200	24.7250	26.7568	31.2641
12	18.5493	21.0261	23.3367	26.2170	28.2995	32.9095
13	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282
14	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233
15	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973
16	23.5418	26.2962	28.8454	31.9999	34.2672	39.2524
17	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902
18	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124
19	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202
20	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147
21	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970
22	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679
23	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282
24	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786
25	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197
26	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520
27	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760
28	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923
29	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012
30	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031
31	41.4217	44.9853	48.2319	52.1914	55.0027	61.0983
63	77.7454	82.5287	86.8296	92.0100	95.6493	103.4424
127	147.8048	154.3015	160.0858	166.9874	171.7961	181.9930
255	284.3359	293.2478	301.1250	310.4574	316.9194	330.5197
511	552.3739	564.6961	575.5298	588.2978	597.0978	615.5149
1023	1081.3794	1098.5208	1113.5334	1131.1587	1143.2653	1168.4972